Effects of Range Doppler-rate Coupling on High Frequency Chirp Radar for Accelerating Targets

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Abstract—Range Doppler coupling is a range bias resulting from the use of linear frequency modulated waveforms. This coupling impacts radar returns of highly accelerating targets resulting in apparent range walk across a processing interval. Because of this, methods for detecting fast objects with long integration times introduce unwanted artefacts to chirp radars utilising low bandwidths and low repetition frequencies. This paper details that in addition to the range bias, range Doppler coupling artefacts impact higher order measurements, and also describes how these affect can be mitigated. Results of radar returns from satellites demonstrating sensitivity improvements are included, illustrating that mitigating range Doppler-rate coupling will benefit any time-frequency method attempting accelerating target detection.

Index Terms—HF radar; radar signal processing; space surveillance

I. INTRODUCTION

HIGH VELOCITY targets, such as those observed by space surveillance radars, present significant challenges for ground based radars, as the required processing interval lengths result in considerable motion throughout the coherent integration time (CIT). This motion, if not matched, can lead to significantly reduced sensitivity. High frequency (HF) radars are well suited to the space surveillance problem: larger wavelengths mean a large receive aperture is achievable with fewer receive channels, fewer channels and low bandwidths result in small data products, and repetitive waveforms and low waveform repetition frequencies (WRF) further ensure this small data product covers all velocities and ranges of interest (albeit ambiguously) [1]. This small nucleus of matched returns enable a large number beams, acceleration processing and long integration times to be processed in real-time. However, a drawback to the use of small bandwidths and low WRFs with linear frequency modulated (LFM) waveforms is the range Doppler coupling artefact, which introduces a velocity-dependant bias to the range measurements [2].

Because the magnitude of the bias is inversely proportional to the chirprate, the impact of range Doppler coupling is generally only of concern for chirp radars utilising low bandwidths and low WRFs. Because of this, range Doppler coupling is often not incorporated into time-frequency methods for matching complex motion, or if it is included, the target’s Doppler across the CIT is treated as being fixed when incorporated into the range Doppler coupling bias [3]–[5].

High velocity target motion throughout a CIT typically degrades sensitivity by causing range-walk and Doppler walk, whereby a target’s return spans multiple range and Doppler cells. Constant velocity range-walk (or indeed radial acceleration range-walk) is typically not a concern for HF radar, due to the coarse range resolution resulting from the use of low bandwidths. However, the large radial accelerations combined with the precise Doppler resolution from the long integration times required typically mean Doppler-walk is the primary cause of degraded sensitivity. This is a well known problem, and HF radars have implemented long-CIT acceleration matching for significant signal to noise ratio (SNR) improvements since the 1960s [6], [7]. However, an aspect that is perhaps not as well appreciated is the complications caused by the range Doppler coupling in conjunction with significant radial acceleration.

Fig. 1. Example range Doppler map showing a satellite spanning several range cells due to Doppler-walk, at around 500 km range. Also present is a large meteor return at around 100 km. The zero-Doppler notch is due to the timeseries-based direct wave and clutter removal. The sensitivity roll-off at large Doppler extents due to the standard Doppler processing approximation (i.e. match at zero Doppler) is also noticeable.

Essentially, with large radial acceleration, the target’s relative velocity changes across the CIT and even if the target’s range lies within one range cell, the measured range may span several range cells due to range Doppler coupling. This will be especially apparent if the acceleration, and the CIT, is large enough that the target’s Doppler span is greater than the WRF. Figure [1] is an example range Doppler map showing a satellite’s returns producing a large Doppler spread, such that range Doppler coupling results in the satellite spanning three range cells. Figure [2] shows several segments of a range Doppler map and the returns of a satellite for increas-
The structure of this paper is as follows: Section II details the slow-time fast-frequency response of an arbitrary waveform, highlighting how range-walk and Doppler-walk present. Section III applies an LFM response to the slow-time fast-frequency expressions showing how the range Doppler coupling combines the acceleration into the range-walk term. Section IV contains illustrative results from a recent space surveillance campaign with an HF line-of-sight radar, demonstrating how mitigating the Doppler-rate coupling improves surveillance campaign with an HF line-of-sight radar, demonstrating how mitigating the Doppler-rate coupling improves sensitivity with long CITs when targets are exhibiting large radial acceleration and jerk. The paper concludes in Section V.

II. STANDARD MIGRATION MITIGATION MEASURES

For a transmitted waveform with match-filtered response \( p(t) \), the baseband compressed signal in the fast-time slow-time domains are approximated by:

\[
S(t, t_k) = A p(t - 2 c^{-1} r(t_k)) e^{-j 2 \pi f_c r(t_k)} ,
\]

where \( t_k \) is the slow time for return index \( k \), \( r(t_k) \) is the range to the target for time instance \( t_k \), \( f_c \) is the transmitted centre frequency, and \( t \) is the fast time such that \( t = t' + T k \) for an overall time \( t' \) and a pulse-interval of \( T \). Also, \( A \) is a constant that depends on the target radar cross section and the target range and \( c \) is the speed of light.

By taking the Fourier transform (FT) over fast-time, the received signal in fast-frequency slow-time is:

\[
S(f, t_k) = A P(f) e^{-j 2 \pi f (f_c + f_v) r(t_k)} ,
\]

where \( P(f) \) is the FT of \( p(t) \).

With a constant velocity motion model across the CIT, \( r(t_k) \) is given by \( r(t_k) = r + v t_k \), and so (2) is now:

\[
S(f, t_k) = A P(f) e^{-j 2 \pi (f + f_v) r + f_v v t_k} ,
\]

where the coupled term \( f_v v t_k \) term represents the linear range migration during the CIT.

There are many methods for mitigating range walk, e.g. the Keystone Transform can be applied to eliminate linear range walk by resampling slow-time as \( t_k = \frac{f_c}{f_v} t' \), where the resulting fast-frequency slow-time expression is:

\[
S(f, t'_k) = A P(f) e^{-j 2 \pi ((f + f_v) r + f_v v t')} ,
\]

which no longer contains the coupling between the fast-frequency, \( f \), and slow-time \( t_k \).

Range-walk is further complicated by accelerating motion. If we take a quadratic motion model, with radial acceleration \( a \) such that \( r(t_k) = r + v t_k + \frac{1}{2} a t_k^2 \), then (2) is now given by:

\[
S(f, t_k) = A P(f) e^{-j 2 \pi ((f + f_v) r + (f + f_v) v t_k + (f + f_v) \frac{1}{2} a t_k^2)} .
\]

Mitigating the range-walk is now challenging because of the inclusion of both linear and non-linear terms \( f_v v t_k \) and \( f a t_k^2 \) respectively. These terms cannot be mitigated with a single slow-time resampling, although there are methods which partially reduce the impact of both \( \[10\] \), \( \[11\] \). However, given that the Doppler walk resulting from accelerating targets is almost always a far more significant degrading factor than for non-accelerating targets, the non-linear range-walk term is typically less of a concern.

The Doppler-walk term is the acceleration-induced slow-time chirp in (5) of \( e^{-j 2 \pi f_v a t_k^2} \). There are many methods applicable to estimating this radial-acceleration, including time-frequency methods, which are crucial for removing this significant source of loss \( \[12\] \)–\( \[16\] \). However, the optimal method is the chirpogram which requires searching through acceleration hypotheses and matching with a slow-time dechirp, allowing an unperturbed Doppler term, \( e^{-j 2 \pi f_v v t_k} \), to be matched by the FT through the coherent integration.

If an acceleration-hypothesis approach is taken, rather than the standard method of applying a slow-time dechirp to the fast-time slow-time product, all non-linear motion terms (i.e. Doppler-walk and range-walk) can be mitigated by applying a phase correction to the fast-frequency slow-time product. Given an acceleration hypothesis \( \alpha \), rather than matching with a standard slow-time dechirp \( e^j 2 \pi f_v a t_k^2 \), the fast-frequency slow-time matching phase matrix \( H_\alpha \):

\[
H_\alpha = e^{j 2 \pi (f + f_v) a t_k^2} ,
\]

which will mitigate all non-linear range and Doppler motion in (5) when \( \alpha = a \). This approach of mitigating the motion in the fast-frequency domain comes at the cost of requiring an additional fast-frequency to fast-time transform prior to the

1Note that the Keystone Transform reference \( \[9\] \), refers to a range and Doppler coupling as being the \( f_v v t_k \) term from \( \[10\] \) and \( \[11\] \); this should not be confused with the use of ‘range Doppler coupling’ in this paper and elsewhere \( \[2\] \).

2Interestingly, the method in \( \[13\] \) essentially utilises range Doppler coupling in reverse in order to estimate the Doppler rate.
standard slow-time to Doppler transform in order to evaluate each acceleration hypothesis.

III. RANGE DOPPLER-RATE COUPLING

Range Doppler coupling is a bias introduced in the measured range for radars operating with LFM waveforms, which is particularly noticeable with low chirp rates. The effect is proportional to the transmit frequency as well as the target velocity, and inversely proportional to the chirp rate. For all systems, this corresponds to a WRF of $p_t = \frac{v}{T_r}$ with $\gamma$ being the chirp rate and $\frac{v}{T_r}$ being the range Doppler coupling constant.

For a transmitted LFM waveform $x(t) = \text{rect}(\frac{t}{T})e^{j\pi \gamma t^2}$, with $T$ being the sweep-width (for an continuous waveform (CW) system, this corresponds to a WRF of $\frac{1}{T}$ Hz), the chirp rate, $\gamma$, is $\frac{f_c}{T}$, with the sign of $\gamma$ specifying the direction, or polarity, of the chirp. Incorporating range Doppler coupling and a very simple chirp model results in (1) and (2) now being given by:

$$S(t, t_k) = A_p(t - 2c^{-1}(r(t_k) + v(t_k)f_c\gamma^{-1}))e^{-j\frac{4\pi}{c}f_c, r(t_k)},$$

$$S(f, t_k) = AP(f)e^{-j\frac{4\pi}{c}((f+f_c)r(t_k))}e^{-j\frac{4\pi}{c}v(t_k)},$$

with $p(t) \approx \text{sin}(Bt)$ and so $P(f) = \text{rect}(\frac{f}{B})$.

The peak of the matched-filter output being delayed by the target’s range as well as its velocity, results in the offending term $e^{-j\frac{4\pi}{c}v(t_k)}$ in (3).

For constant velocity motion (that is $r(t_k) = r + vt_k$ and $v(t_k) = v$), this offending term is $e^{-j\frac{4\pi}{c}v t_k}$ and is not slow-time dependant. This term represents the constant time-delay shift due to the coupling. However, with radial acceleration, the result is more complicated. If $r(t_k) = r + vt_k + \frac{1}{2}at_k^2$ and $v(t_k) = v + at_k$, is now given by:

$$S(f, t_k) = AP(f)e^{-j\frac{4\pi}{c}((f+f_c)r+\frac{1}{2}v t_k+(f+f_c)v+\frac{1}{2}at_k^2) t_k} \times e^{-j\frac{4\pi}{c}((f+f_c)at_k^2)}.$$  \hspace{1cm} (9)

The previous expression for the linear range walk term in (3) was simply $f_v t_k$, however now in (9), the range walk term is:

$$f \left( v + \frac{f_c}{\gamma} a \right) t_k.$$  \hspace{1cm} (10)

That is, due to inclusion of range Doppler coupling, combined with an accelerating target, instead of the linear range walk being solely proportional to the velocity $v$, it is now proportional to $v + \frac{f_c}{\gamma} a$. The radial acceleration contributes to linear range walk, as well as linear Doppler walk and non-linear range walk! In fact, depending on the specific parameters and accelerations, $\frac{f_c}{\gamma} a$ may well be significant. By way of example, for the results in Section IV the range Doppler coupling constant, $\frac{f_c}{\gamma}$, is 32.45 s. Also, at the point of a target’s closest approach, the radial velocity will be zero and the radial acceleration will be at its greatest, meaning that there will be significant apparent range-walk despite there being no (instantaneous) velocity.

The problem with this apparent range walk, beyond the loss in sensitivity due to the range walk, is that the use of long-CIT integration methods to increase coherent gain will not be effective without taking into account the impact of the coupling. Adjustments can be made to the matching fast-frequency slow-time products to incorporate the full effect of the range Doppler coupling in order to minimise loss. However, with periodic waveforms the ambiguous velocity returns are a further complication.

As mentioned in Section II, a significant benefit to the ambiguous velocity returns with chirp/periodic waveforms is the entire target velocity range of interest being sampled by processing only the ambiguous region, essentially undersampling Doppler. This greatly benefits long-CIT processing, especially with a large acceleration search. However, a downside to these ambiguous returns (beyond the uncertainty in a detection’s true Doppler) is that any time-frequency mitigation method, including those detailed in Section II, needs to search this ambiguous velocity space. This searching breaks one of the main benefits of having small data rates from ambiguous velocity returns, as now a potentially large unambiguous velocity span needs to be essentially recreated.

If $M$ represents the expected number of Doppler ambiguities (also known as the fold factor), such that the Doppler centre of an individual delay-Doppler map is given by $\frac{j\pi}{T}$ Hz, then velocity term in (9) is now replaced by $v = \frac{\nu_{amb}}{M} + \frac{j\pi}{2T}$ Hz, where $\nu_{amb}$ is the ambiguous, or blind, velocity (still matched by the FT) and lies between $\pm \frac{\nu_{amb}}{M}$ m/s.

Incorporating all these aspects and fully matching for the motion given the impacts of the range Doppler coupling, (9) is now given by:

$$H_{\alpha, M} = e^{j\frac{4\pi}{c}((f+f_c)M^{-1} + \frac{f_c}{j\pi M} \alpha) t_k + \frac{1}{2}(f+f_c)at_k^2).$$  \hspace{1cm} (11)

This now requires searching across the acceleration region of interest as well as ambiguous velocity regions ($\alpha$ and $M$ respectively). However, all the impacts of acceleration are fully matched, both across the fast-frequency and Doppler domains.

Other time-frequency methods can be applied as they require searching across ambiguous velocity regions, for example if the Keystone Transform was applied to the slow-time fast-frequency product, then (11) is able to be modified such that:

$$H_{\alpha, M'} = e^{j\frac{4\pi}{c}((\frac{\nu_{amb}}{M} + \frac{f_c}{j\pi M} \alpha) t_k + \frac{1}{2}(f+f_c)at_k^2).}$$ \hspace{1cm} (12)

This approach would now allow other methods to be applied, an example being the half-blind velocity search which additionally adjusts the slow-time fast-frequency product by $\frac{1}{M}$ Hz for each CIT [17], [18], as any returns which straddle across two ambiguous regions will be fully contained within a single region after being shifted by half the WRF. Although, rather than directly shifting everything by $\frac{1}{M}$ Hz, one half of the $M$th product would be combined with the other half of the $(M + 1)$th product, and so on for additional steps.

Despite applying these matching products, the full extent of the apparent range walk (such as the cases shown in Figure 2) will not be completely mitigated, as only a single WRF Hz segment is ever processed. These methods will assist in matching the motion across each segment, however they do not cover segments of the target’s Doppler return which extend beyond a single ambiguous WRF region.
A. Jerking Chirps

Although the radial range acceleration is the primary cause of loss of sensitivity for high speed target detection, target motion is typically not constant across a CIT and so additional processing to include jerk as well as acceleration, or indeed an arbitrary range profile, can improve sensitivity [19], [20]. However, range Doppler coupling also impacts the higher order range walk terms in the same manner, in that it conflates linear range walk with the radial acceleration.

If we take the jerk to be \( \ddot{a} \), such that \( r(t_k) = r + vt_k + \frac{1}{2}at_k^2 + \frac{1}{6}\ddot{a}t_k^3 \) and \( v(t_k) = v + at_k + \frac{1}{2}\dot{a}t_k^2 \), then (9) becomes:

\[
S(f, t_k) = AP(f) e^{-j \frac{\pi}{2} \left( (f + f_c)r + \frac{f_c}{c}v + ((f + f_c)v + \frac{f_c}{c}\ddot{a})t_k \right) - j \frac{\pi}{4} \left( \frac{1}{2}(f + f_c)\ddot{a}t_k^2 + \frac{1}{6}(f + f_c)\dot{a}t_k^3 \right)}.
\]

The linear range-walk term is still impacted by range Doppler coupling, with the term \( \frac{f_c}{c} \) folded in. But now the second-order range-walk terms are similarly impacted by the third order range rate. The expression in (11) can be extended to form the matching term to mitigate all first and second order range and Doppler walk, as well as the third order Doppler walk:

\[
H_{\alpha, \dot{a}, M} = e^{j \frac{\pi}{4} \left( (f + f_c)\frac{f_c}{c}v + \frac{f_c}{c}\ddot{a}t_k + \frac{1}{2}(f + f_c)v + \frac{1}{2}(f + f_c)\ddot{a}t_k^2 \right) - j \frac{\pi}{4} \left( \frac{1}{6}(f + f_c)\dot{a}t_k^3 \right)}.
\]

This approach can be extended to an arbitrary number of matching terms, with the range-walk resulting from one term also being impacted by its subsequent rate, multiplied by the range Doppler coupling factor.

IV. RESULTS

The results in this section were collected during a trial deployment of Defence Science and Technology Group’s (DSTG) High Frequency Line of Sight (HFLOS) radar. The HFLOS radar is an experimental HF wide area surveillance staring radar, enabling rapid prototyping and evaluation of improvements to every aspect of the system [21]–[24]. In recent years, DSTG has undertaken a series of space surveillance test deployments of the HFLOS radar. A full description of these campaigns will be detailed in a future publication. The results in this section are from 2020, using a quasi-monostatic configuration with two dual-polarised transmit arrays located several kilometres away from a thirty-element linear receive array of horizontal dipole doublets, as shown in Fig. 3.

The transmitted waveform is a LFM CW with a centre frequency of 32.45 MHz, a bandwidth of 10 kHz and a WRF of 100 Hz. These parameters correspond to a chirprate of 1 MHz s\(^{-1}\) (therefore, an upchirp) and a range Doppler coupling factor of 32.45 s.

Typical HFLOS radar processing consists of a 10.24 s CIT, with a 0.5 s stride time, several hundred finger beams and a large Doppler-rate search range. For this paper’s results, which includes some extreme processing lengths, in order to evaluate long CIT processing, a moving surveillance beam was formed following the satellite throughout the processing interval. The knowledge of the satellite location was based on the published SpaceTrack two-line elements and propagated using the Simplified General Perturbations model, Version 4. This ensured there was no loss of sensitivity due to the motion migrating through static surveillance beams. Although such an approach is not generally applicable for typical targets with unknown trajectory, for the purposes of this paper it isolates and highlights the relative losses caused by accelerating (and jerking) motion.

Figure 4 shows the SNR of a Starlink satellite at a point of significant radial acceleration (73.72 m/s\(^2\) instantaneous) for various CIT lengths, comparing the standard acceleration matching dechirp compared with the time-frequency method attempting to full match the acceleration given by (11). Incorporating the range Doppler-rate coupling mitigation allows longer CITs to be used, resulting in improved sensitivity.

Interestingly the (instantaneous) radial jerk and snap are \(-0.433 \text{ m/s}^3\) and \(-0.221 \text{ m/s}^4\) respectively, which suggests that higher order approaches would be required for such long CIT lengths [25].

Fig. 4. SNR returns of a Starlink satellite utilising the standard dechirp method in comparison with the matching method outlined in (11).

A more extreme motion example is a CZ-11 rocket body at low altitude prior to imminent reentry, 302 km range from the radar. The close approach ensures larger radial rates as shown in Figure 5 with acceleration reaching 172 m/s\(^2\) and the jerk ranging between \(\pm 4 \text{ m/s}^3\).

Fig. 5 shows the SNR of the rocket body, processed with a 10.24 s CIT and comparing the acceleration-only dechirp matching, acceleration and jerk matching as well as the full acceleration and jerk mitigation with (14). For the first half of the pass, where the velocity is in the opposite direction to the acceleration and the range Doppler coupling factor, the full processing is similar to the jerk-matched processing.
However, for the second half of the pass, the acceleration and jerk matched process does not perform any better than the acceleration only process, and the full range Doppler-rate coupling matched process improves sensitivity up to 5 dB.

Beyond the sensitivity improvements, Figure 7 shows the range time returns from the three methods for the rocket body pass. The acceleration subplot as well as the acceleration and jerk subplot show considerable range spread during the second half of the pass. However the third subplot, with the range Doppler-rate coupling matched process, generally exhibits the maximum sensitivity in a more well-defined peak. The energy is still spread over a significant number of range cells (again, as the process is under sampling Doppler it can only match a single WRF Hz segment), but the peak is now more concentrated.

These results have been generated using the standard CIT length of 10.24 s. However, Figure 8 shows the same three methods used in Figure 7 across a full CIT range from 1.28 s to 30.72 s. The third subplot (utilising the matching process of (14)) illustrates that the modest sensitivity improvements occurring during the second half of the target’s pass are consistent across all CITs. This mirrors the similar gains shown in Figure 4.

The rocket body motion outlined in Figure 5 indicates that all the motion parameters are changing rapidly during the CIT. This, and the non-zero and changing jerk from the the Starlink pass from Figure 4 suggest that more motion parameters, such as matching the snap, would be needed to fully match the motion. However, the ambiguous velocity returns essentially undersample Doppler, so the process can only ever match a single WRF Hz segment. Because of this, such long CITs will not be able to fully match all returned energy from such accelerating targets.

V. Conclusion

Range Doppler coupling is a range bias resulting from the use of LFM waveforms, and with accelerating targets, this bias leads to apparent range-walk. As a result, time-frequency methods will not be applicable to radars utilising low-chirp-rate LFM waveforms. By mitigating these effects, time-frequency processing can better match the motion for SNR gains, including with accelerating and jerk motion. For a typical satellite as well as a rocket body with extreme motion, sensitivity improvements up to 5 dB have been demonstrated. These gains have been illustrated with only a very simple chirp model, and a more accurate approach may yield better results [26]. For significantly accelerating and jerking targets, long CIT processing can result in a Doppler span greater than WRF Hz. This Doppler span will impose a hard limit on the achievable maximum CIT lengths whilst undersampling Doppler.

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Fig. 7. The full range time plots of the rocket body pass corresponding to the SNRs in Figure 8. The top subplot is the acceleration dechirping, the middle subplot is the acceleration and jerk dechirping, the final subplot is the full range Doppler rate coupling mitigation, from Eq. (14).

Fig. 8. SNR of a pass of a rocket body, with a wide range of CITs, processed with traditional acceleration dechirping, acceleration and jerk dechirping, and finally acceleration and jerk range Doppler-rate coupling mitigation, from Eq. (14).


